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Research Article

Physics-informed neural networks (PINNs) for inverse analysis: An illustration

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ABSTRACT

This paper investigates the application of physics-informed neural networks (PINNs) to solve inverse problems in solid mechanics. Traditional numerical methods, such as finite element method (FEM) and isogeometric analysis (IGA), often require significant computational time, hindering their applicability in scenarios requiring rapid calculations. To overcome these challenges, PINNs offer a robust alternative by directly incorporating physical information—such as governing equations and boundary conditions—into the neural network's loss function, reducing the reliance on purely experimental data. This study specifically focuses on a two-dimensional linear elasticity plane-strain problem defined on a unit square domain. The primary objective is to accurately determine the spatial distribution of Young's modulus, alongside displacement and stress components, based on established governing equations, boundary conditions, and sparsely measured data. To achieve this, the proposed framework employs two distinct neural networks: the first ($\mathcal{N}_1$) is designed to predict displacements, while the second ($\mathcal{N}_2$) estimates Young's modulus. Both network architectures consist of four hidden layers containing 40 neurons each. The models were trained over 20,000 epochs, utilizing a dataset comprising displacements measured at 400 points and Young's modulus measured at 9 points randomly distributed across the computational domain. The results demonstrate a strong agreement between the PINNs predictions and the exact analytical solutions. Although slight relative errors were observed at the boundaries where exact values approach zero, the overall deviation remained acceptably small, at only around 10^{-5}. These findings validate that PINNs provide a highly effective tool that balances computational efficiency and error. While the study highlights a current lack of neural network architecture optimization, systematically addressing this in future research will further enhance the model's speed and predictive accuracy for complex mechanical systems.

Key words: Physics-informed Neural Networks, PINNs, inverse mechanical problems, inverse plane strain problem

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INTRODUCTION

There are two primary types of problems in mechanics: forward and inverse. Forward problems involve determining the behavior of structures based on known forces, boundary conditions, and material properties. This type of problem is commonly applied in structural design. Conversely, inverse problems utilize observed structural behavior from experiments to determine unknown values, such as material properties or distributions. Inverse problems, particularly those related to material distribution, have significant applications in diagnosing and monitoring infrastructure health.

Various methods have been developed to address inverse mechanical problems, including finite element methods (FEM), isogeometric analysis (IGA), and experimental techniques. Several studies, such as those by Nguyen–Thanh et al. [1–4], have explored the application of IGA and FEM to solve a wide range of solid mechanics problems. These studies aim to leverage the strengths of IGA and FEM in accurately representing complex geometries and analyzing the mechanical behavior of solid structures. Through these numerical methods, researchers have contributed substantially to understanding and addressing challenges in solid mechanics.

However, a notable disadvantage of these methods is the significant computational time required. The complexity of solid mechanics problems and the need for detailed analysis of large-scale structures often lead to lengthy computations. This limitation can hinder their applicability in real-time scenarios or cases requiring rapid calculations, particularly in dynamic phenomena. Researchers are actively investigating parallel computing, domain decomposition, and reduced-order modeling to overcome this challenge. Additionally, hybrid approaches that combine IGA or FEM with other methods are being developed

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to enable more efficient analysis of complex solid mechanics problems.

Deep learning has emerged as a promising approach to solving inverse problems across various fields in recent years. Artificial neural networks (ANNs) have been widely adopted in engineering disciplines, including material science, fluid mechanics, and infrastructure health monitoring. These ANNs, often called "data-driven" models, are typically trained using data from experiments, numerical simulations, or exact solutions. However, a particular class of ANNs known as physics-informed neural networks (PINNs) offers a different approach. Rather than relying solely on data, PINNs incorporate physical information directly into the training process. The loss function of PINNs integrates governing equations, initial and boundary conditions, and other relevant constraints, making the model aware of the underlying physics.

Several studies have explored the application of PINNs in various solid mechanics problems, elasticity imaging, and material identification. Haghighat et al.[5] proposed a physics-informed deep learning framework for inversion and surrogate modeling in solid mechanics, leveraging PINNs to estimate material properties and predict the behavior of solid materials based on measured data and physical laws. Kamali et al.[6] utilized PINNs in elasticity imaging to discover the spatial distribution of elastic modulus and Poisson's ratio, achieving accurate and efficient material property estimation by integrating physics-based constraints into the neural network architecture. Zhang et al.[7] focused on nonhomogeneous material identification in elasticity imaging using PINNs, demonstrating the effectiveness of this approach in characterizing the spatial distribution of material properties and identifying complex material behavior. Chen and Gu[8] explored deep neural networks in learning hidden elasticity, showing how PINNs can capture complex nonlinear elastic behavior and uncover hidden patterns in material responses. In another study, Chen and Gu[9] extended the application of PINNs to various elasticity problems, including forward, inverse, and mixed problems, highlighting the versatility of PINNs in addressing a broad range of mechanical challenges and their potential for efficient and accurate simulations.

These studies underscore the potential of PINNs in solid mechanics and elasticity-related research. By integrating deep learning techniques with physical principles, PINNs provide a powerful tool for solving inverse problems, characterizing material behavior, and predicting mechanical responses[10–17]. Incorporating PINNs into mechanics opens new avenues for efficient and accurate analysis, reducing reliance on traditional experimental approaches.

This work focuses on applying PINNs to solve a specific problem: the 2D linear elasticity problem. The objective is to determine displacements, stress components, and Young's modulus within the computational domain based on the governing equations, boundary conditions, body forces, and displacements measured at specific locations. By employing PINNs, we aim to harness the power of deep learning, combined with the inherent physics embedded in the governing equations, to estimate the desired quantities accurately. The unique training approach of PINNs allows for a more physics-aware model, potentially offering improved solutions compared to traditional data-driven ANN models.

THEORIES

Analytical solution

To demonstrate the application of PINNs, we consider an elastic plane-strain problem defined on a unit square, as depicted in Figure 1. The goal is to analyze the behavior of the elastic material within this unit square. To solve this problem using PINNs, we must define the governing equation, boundary conditions, and any other relevant constraints. The specific boundary conditions for this problem are shown in Figure 1.

By applying the PINNs model to this problem, we aim to predict the displacements, stress components, and Young's modulus within the unit square. The PINNs model, trained using the governing equation, boundary conditions, and additional constraints, will provide accurate estimates of these quantities.

The body forces are defined as:

$$
\begin{aligned}
F_x = & -\frac{E}{v^2-1}\left(4\pi^2\sin(\pi y)\left(2\cos(\pi x)^2-1\right)-4vy^2\pi\cos(\pi x)\right) \\
& -\frac{E}{2(v+1)}\left(4y^3\pi\cos(\pi x)-\pi^2\sin(\pi x)\left(2\cos(x\pi)^2-1\right)\right) \\
F_y = & \frac{E\left(12y^2\sin(\pi x)-2v\pi\cos(\pi x)\sin(\pi x)\right)}{v^2-1} \\
& +\frac{E\pi^2\left(2\cos(\pi y)\ \sin(2\pi x)+y^4\sin(\pi x)\right)}{2(v+1)}
\end{aligned}
\tag{01}
$$

The exact solution[5] to this problem

$$
\begin{aligned}
u_x &= \cos(2\pi x)\sin(\pi y), \\
u_y &= \sin(\pi x)\, y^4
\end{aligned}
\tag{02}
$$

The governing equations:

$$
\begin{aligned}
&\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\tau_{xy}}{\partial_y}+F_x=0, \\
&\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial_y}+F_y=0
\end{aligned}
\tag{03}
$$

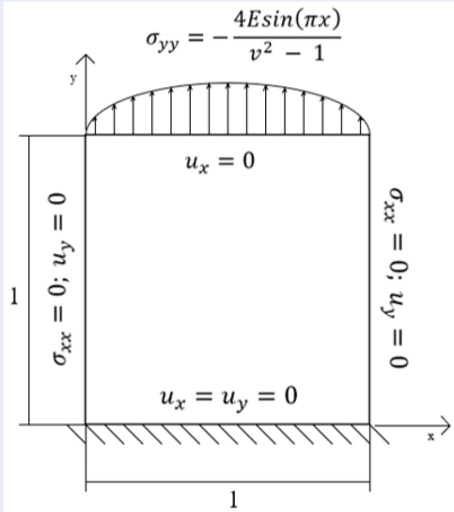

Figure 1: Problem setup andboundary conditions (Source: Authors)

where σ_{xx}, σ_{yy}, τ_{xy} are the stress components. F_x and F_y are, respectively, the body forces along the x and y axes.

Strain – stress relationships by Hooke's Law:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-v^2} \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1-v}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \quad (04)$$

where E is Young's Modulus; v is a Poisson's ratio and ε_{xx}, ε_{yy}, γ_{xy} are strain components defined as

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}, \ \varepsilon_{yy} = \frac{\partial u_y}{\partial y}, \\ \gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \quad (05)$$

where u_x and u_y are the displacement components along the x and y axes, respectively.

In this case, the Poisson's ratio v is 0.3, and Young's Modulus is given as

$$E = 1 + x^2 + y^2 \quad (06)$$

PINNs model

In this problem, two neural networks are employed to address the task. The first neural network, denoted as $\mathcal{N}_1$, is used to predict displacement. As shown in Figure 2, $\mathcal{N}_1$ has an input layer with two neurons that receive the coordinate values within the computational domain. This enables the network to capture the spatial information necessary for displacement estimation. The network is then constructed with four hidden layers containing 40 neurons, which help extract and learn complex patterns and relationships from the input data. Finally, $\mathcal{N}_1$ includes an output layer with two neurons that provide the predicted displacement values based on the learned patterns within the network.

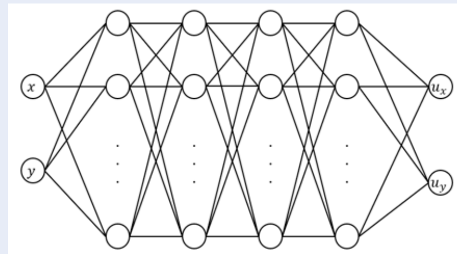

Figure 2: The neural networkfor displacements ($\mathcal{N}_1$) (Source: Authors)

The second neural network, $\mathcal{N}_2$, is employed to predict Young's modulus in this problem. The architecture of $\mathcal{N}_2$ is similar to that of $\mathcal{N}_1$ with a few key differences. As illustrated in Figure 3, $\mathcal{N}_2$ begins with an input layer of two neurons, which receive the coordinate values within the computational domain. These input neurons allow the network to incorporate spatial information into the prediction process. Like $\mathcal{N}_1$, $\mathcal{N}_2$ also consists of four hidden layers, each with 40 neurons. Nonetheless, unlike $\mathcal{N}_1$ the output layer of $\mathcal{N}_2$ has a single neuron that provides the predicted value of Young's modulus. This model is trained over 20,000 epochs. The stress components are then computed from the predicted displacement using Eqs. Equation (04) andEquation (05).

This model uses the activation function for the network $\mathcal{N}_1$and the softplus activation function for the network $\mathcal{N}_2$. The optimization algorithm used for this model is FMINCON from MATLAB.

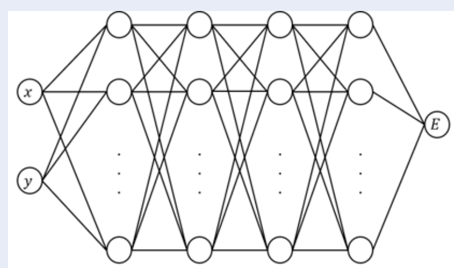

Figure 3: Neural network forYoung's modulus ($\mathcal{N}_2$) (Source: Authors)

The loss function of this model is defined as

$$L = L_{GE} + L_{BC} \quad (07)$$

where L_{GE}, L_{CB} are loss components from the governing equation, boundary conditions. The governing equation loss, L_{GE}, is determined as

$$L_{GE} = \left(\frac{\partial \sigma^*_{xx}}{\partial x} + \frac{\partial \tau^*_{xy}}{\partial y} + F_x\right)^2 + \left(\frac{\partial \tau^*_{xy}}{\partial x} + \frac{\partial \sigma^*_{yy}}{\partial y} + F_y\right)^2$$

Where σ^*_{xx}, σ^*_{yy}, τ^*_{xy} are the predicted stress components values calculated over the full domain.
The boundary condition loss, L_{BC}, is defined as

$$L_{BC} = \sum_i^{N_{BC}} \left(f^*_{BC_i} - f_{BC_i}\right)^2 \qquad (09)$$

Where N_{BC} is the number of control points on the boundary, along with some points where the values (displacement and Young's modulus) are already measured.
In this context, we consider a computational domain with a grid of 100x100 points. The boundary conditions are illustrated in Figure 1. Due to the multimodal nature of an inverse problem, we need additional displacement and Young's modulus values at several points to obtain accurate results. We measured displacement at 400 points and Young's modulus at 9 points. These points are distributed randomly across the computational domain.

RESULTS

Results obtained for displacement components u_x and u_y are, respectively, exhibited in Figure 4 and Figure 6. In addition, comparisons for the stress components are shown in Figures 8 and 10, and Figure 12. Eventually, distributions of Young's modulus are plotted in Figure 14.
The error of displacement, stress components, and Young's Modulus (%) is illustrated in Figures 5, 7, 9, 11 and 13 and Figure 15. In all the figures mentioned, the good agreement between the neural network predictions and the exact solutions is evident, i.e., the relative difference between the predicted and exact values is acceptably small.
The error of the predicted results with the exact results is defined as:

$$\varepsilon = \frac{\left\|v_{exact} - v_{predict}\right\|_2}{\left\|v_{exact}\right\|_2} \qquad (10)$$

Where v_{exact} is the exact value of the problems from Eqs. Equations (02), (03), (04), (05) and (06); $v_{predict}$ is the predicted value from 2 neural networks $\mathcal{N}_1$ and $\mathcal{N}_2$.
The relative errors are slightly higher at the boundary values, especially where the displacement or stress components are zero. This is because we are comparing the predictions with analytical solutions. Since

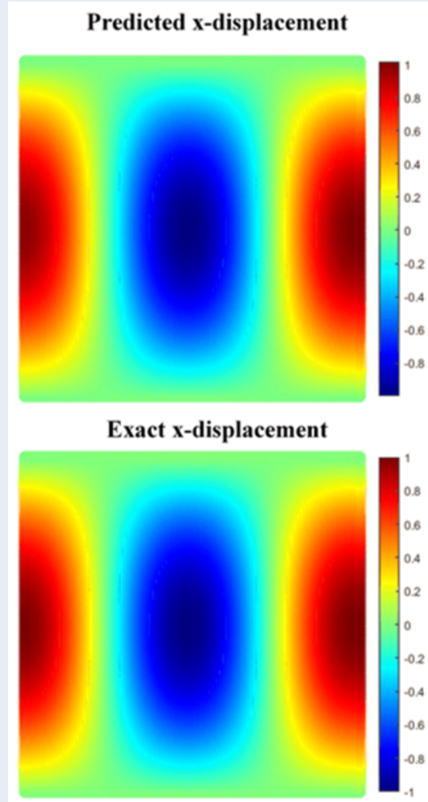

Figure 4: Result of u_x (Source: Authors)

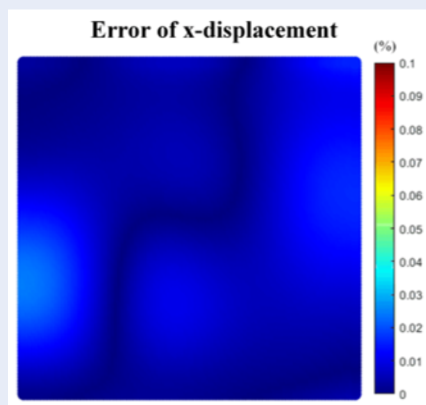

Figure 5: Error of u_x (Source: Authors)

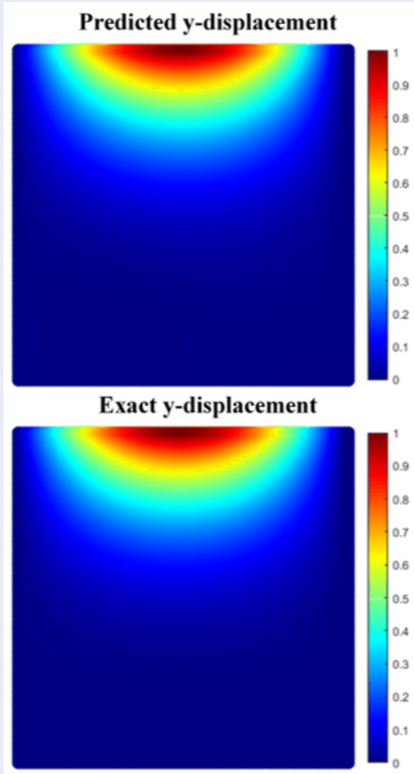

Figure 6: Result of u_y (Source: Authors)

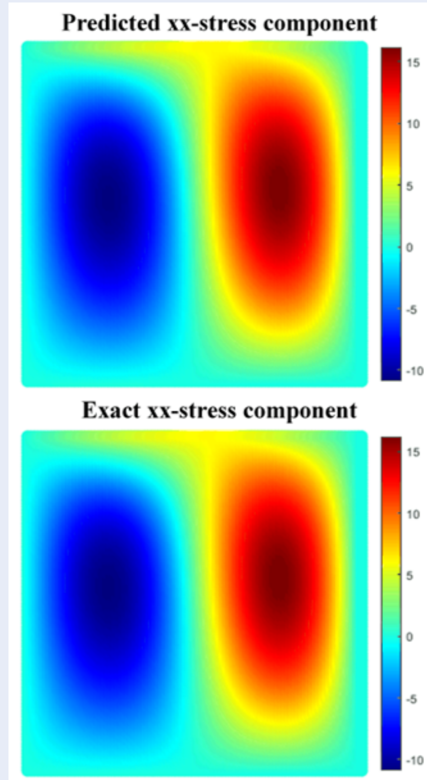

Figure 8: Result of σ_{xx} (Source: Authors)

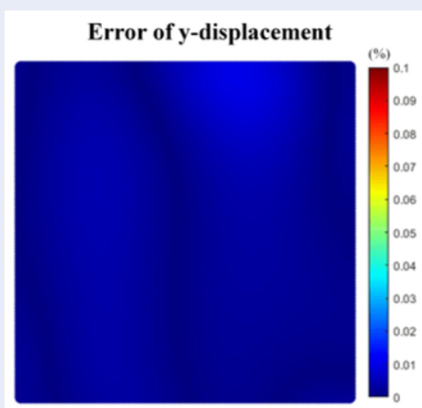

Figure 7: Error of u_y (Source: Authors)

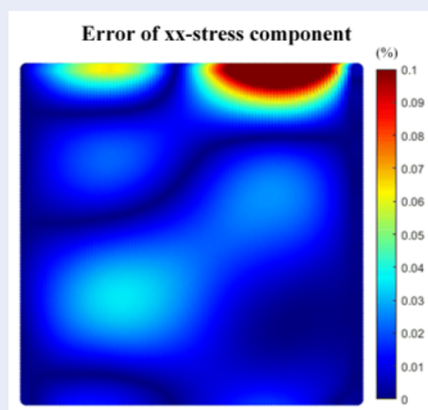

Figure 9: Error of σ_{xx} (Source: Authors)

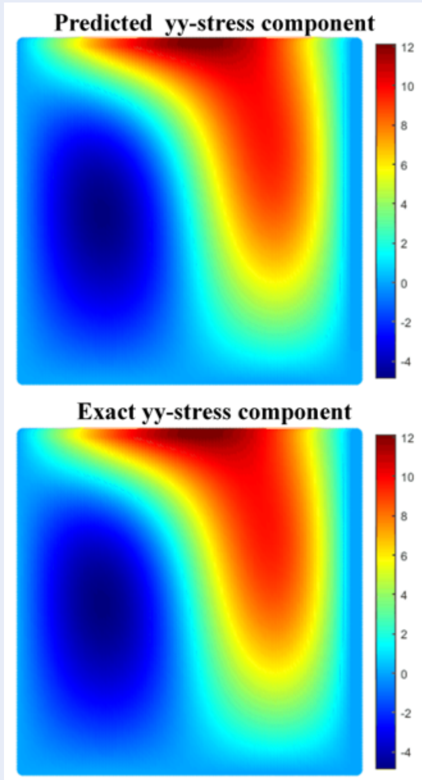

Figure 10: Result of σ_{yy} (Source: Authors)

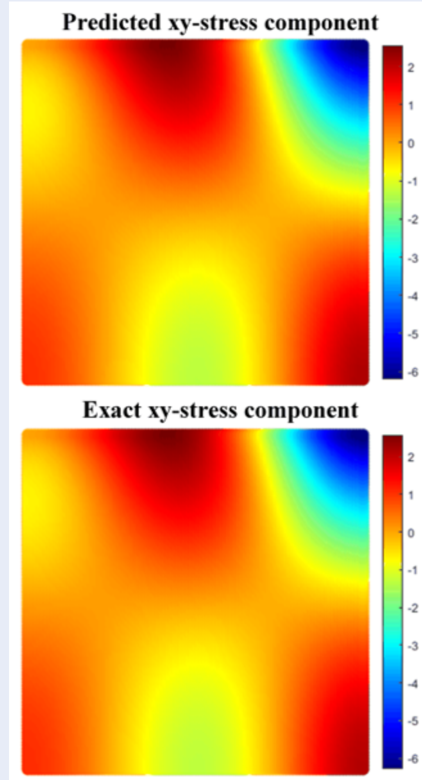

Figure 12: Result of τ_{xy} (Source: Authors)

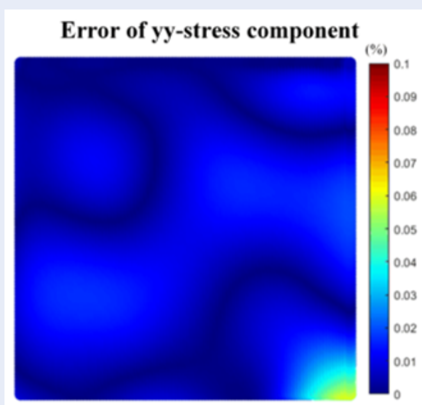

Figure 11: Error of σ_{yy} (Source: Authors)

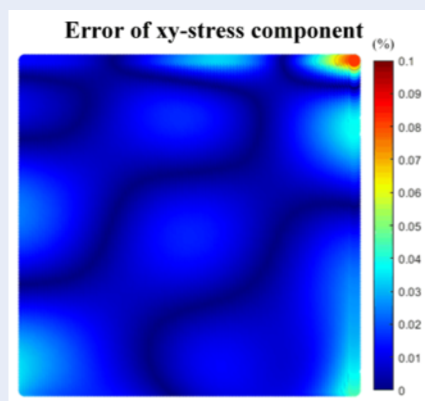

Figure 13: Error of τ_{xy} (Source: Authors)

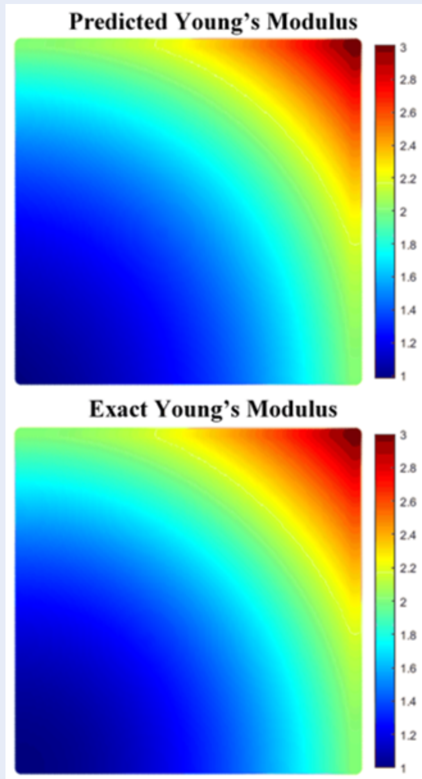

Figure 14: Result of Young's Modulus (Source: Authors)

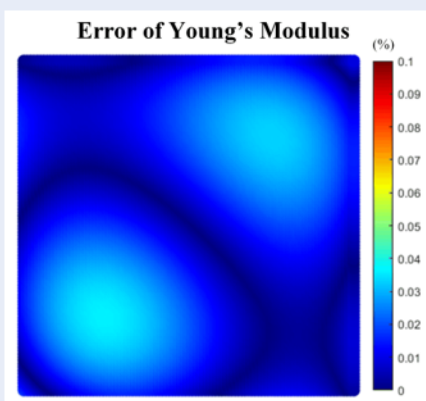

Figure 15: Error of Young's Modulus (Source: Authors)

PINNs are an approximation method, the predicted results will naturally have some error. The relative error can be more significant when the exact solution at a prediction point is zero. However, even though the relative errors at the boundaries are relatively high, the results from the PINNs are still acceptable, with the deviation between the predicted and exact values being only around This shows that, despite some limitations, the overall performance of the PINNs remains reliable for practical applications.

These results demonstrate that PINNs can achieve performance close to the exact solutions, validating their effectiveness as a computational tool in solid mechanics. Given the minor errors, PINNs can be considered reliable for predicting mechanical properties and behaviors in various engineering applications, especially in scenarios where traditional methods may be computationally expensive or infeasible. Thus, the results illustrated that PINNs strike a balance between computational efficiency and error, reinforcing their potential as powerful tools for solving complex problems in engineering and applied mechanics.

CONCLUSIONS

This study demonstrates the application of physics-informed neural networks (PINNs) for inverse analysis in mechanics, specifically within a unit-square domain under plane-strain conditions. The problem setup includes defined geometry, loading, boundary conditions, and displacement measurements at specific locations. The critical challenge addressed is the unknown distribution of Young's modulus. To tackle this, two neural networks are designed: one to predict displacements and the other to estimate Young's modulus. These networks are trained by minimizing a loss function incorporating the governing equations, boundary conditions, and the discrepancy between exact and predicted displacements. The error of the PINNs approach is validated through the strong agreement between the expected results and analytical solutions for various quantities, including displacements, stress components, and Young's modulus. A notable advantage of PINNs is their ability to handle both forward and inverse analysis problems with a unified approach. This capability opens new avenues for solving inverse problems more effectively and efficiently, potentially transforming the field and enabling practical applications across various domains.

However, this study has shortcomings, including a lack of optimization of the neural network architecture. This limitation means the model may take longer to train and may not fully realize its potential for accurate predictions. The network might struggle to learn complex patterns effectively without a suitable architecture, leading to suboptimal performance. Consequently, addressing this issue through systematic optimization of the neural network architecture could enhance the model's speed and accuracy, enabling more efficient solutions to the problems under investigation. Future research should focus on fine-tuning these architectural parameters to achieve better overall performance.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

AUTHOR CONTRIBUTION

Nhi Phuong Thi Nguyen: Data curation, Validation, Writing-original draft.
Minh Ngoc Nguyen: Formal analysis, Visualization.
Tinh Quoc Bui: Investigation, Project administration.
Duy Vo: Conceptualization, Methodology, Writing-review & editing.
Khuong D. Nguyen: Supervision, Writing-review & editing.

NOMENCLATURE

PINNs: physics-informed neural networks
FEM: finite element method
IGA: Isogeometric analysis
ANNs: Artificial neural networks

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Bài nghiên cứu

Minh họa về việc sử dụng mạng nơ-ron nhân tạo kết hợp thông tin vật lý (PINNs) cho bài toán nghịch trong cơ học

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TÓM TẮT

Bài báo này nghiên cứu việc áp dụng mạng nơ-ron nhân tạo kết hợp thông tin vật lý (PINNs) để giải quyết các bài toán ngược trong cơ học vật rắn. Các phương pháp số truyền thống, chẳng hạn như phương pháp phần tử hữu hạn (FEM) và phân tích đẳng hình học (IGA), thường đòi hỏi thời gian tính toán đáng kể, làm cản trở khả năng ứng dụng của chúng trong các tình huống yêu cầu tính toán nhanh. Để vượt qua những thách thức này, PINNs cung cấp một giải pháp thay thế mạnh mẽ bằng cách tích hợp trực tiếp thông tin vật lý - chẳng hạn như các phương trình chủ đạo và điều kiện biên - vào hàm mất mát của mạng nơ-ron, giúp giảm sự phụ thuộc vào dữ liệu thuần thực nghiệm. Nghiên cứu này đặc biệt tập trung vào bài toán biến dạng phẳng đàn hồi tuyến tính hai chiều được xác định trên một miền hình vuông đơn vị. Mục tiêu chính là xác định chính xác sự phân bố trong không gian của mô-đun Young, cùng với các thành phần chuyển vị và ứng suất, dựa trên các phương trình chủ đạo đã thiết lập, điều kiện biên và các dữ liệu được đo đạc thưa thớt. Để đạt được điều này, hệ thống đề xuất sử dụng hai mạng nơ-ron riêng biệt: mạng thứ nhất ($\mathcal{N}_1$) được thiết kế để dự đoán các chuyển vị, trong khi mạng thứ hai ($\mathcal{N}_2$) ước lượng mô-đun Young. Cả hai kiến trúc mạng đều bao gồm bốn lớp ẩn, mỗi lớp chứa 40 nơ-ron. Các mô hình đã được huấn luyện qua 20.000 lần lặp, sử dụng tập dữ liệu bao gồm các chuyển vị được đo tại 400 điểm và mô-đun Young được đo tại 9 điểm phân bố ngẫu nhiên trên toàn bộ miền tính toán. Các kết quả chứng minh sự đồng thuận cao giữa các dự đoán của PINNs và các nghiệm giải tích chính xác. Mặc dù có quan sát thấy những sai số tương đối nhỏ ở các biên nơi các giá trị chính xác tiến gần đến 0, độ lệch tổng thể vẫn ở mức nhỏ chấp nhận được, chỉ vào khoảng 10^{-5}. Những phát hiện này xác nhận rằng PINNs cung cấp một công cụ cực kỳ hiệu quả, cân bằng tốt giữa hiệu suất tính toán và sai số. Mặc dù nghiên cứu chỉ ra sự thiếu hụt hiện tại trong việc tối ưu hóa kiến trúc mạng nơ-ron, nhưng việc giải quyết vấn đề này một cách có hệ thống trong các nghiên cứu tương lai sẽ nâng cao hơn nữa tốc độ và độ chính xác dự đoán của mô hình đối với các hệ cơ học phức tạp.

Từ khoá: Mạng nơ-ron nhân tạo kết hợp thông tin vật lý, PINNs, các bài toán ngược trong cơ học, bài toán ngược biến dạng phẳng

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